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CALCULATION OF AN APPROXIMATE COOLING CURVE FOR COOKED ROAST BEEF[1]

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ABSTRACT

On June 1, 1983, the Food Safety and Inspection Service of the U.S. Department of Agriculture issued a final regulation regarding cooking and cooling requirements for roast beef. This regulation includes a specification of the time that the cooked roast may be exposed to temperatures in the range 120°F (332K) and 55°F (285.93K), no more than 6 h, and that chilling must continue until the center temperature reaches 40°F (277.6K) before packaging. The purpose of this work was to suggest means by which processors may estimate these temperature limits. Several measured temperature profiles at the center of the roasts were obtained from beef processors. It was found that the measured temperatures obtained during the cooling of cooked roasts by ice water, may be predicted satisfactorily by methods described in this paper. Cooling with cold air introduces a boundary layer resistance to heat transfer and possibly also mass transfer because of potential evaporation of water. However, in this latter case further investigation is needed to determine its effect on the cooling of moist food products; the use of heat transfer considerations alone gave reasonable agreement with measurement in this particular instance. It appears that ice water velocities should be kept in excess of about 0.076 m/s to ensure that minimal surface resistance to heat transfer occurs during cooling; ice water velocities in the order of 0.15 m/s seem to lead to excellent cooling conditions.

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INTRODUCTION

In the industrial processing of roast beef, the roast beef is cooked and chilled under prescribed conditions determined by quality considerations and health safety regulations. The Food Safety and Inspection Service of the U.S. Department of Agriculture published a final rule in the Federal Register (Vol. 48, No. 106, June 1, 1983) which established production requirements for roast beef cooking and chilling. This final rule specifies that after the cooking cycle is completed, the maximum chilling time is "restricted to no more than 6 h for the time that the produce is in the optimal growth zone for pathogens, the zone between 120 °F (322.40K) and 55 °F (285.93K), with the stipulation that chilling then continues until 40 °F (277.6K) is reached and that the product is not packed for shipment until this occurs." Clearly, the subsequent cooling cycle after cooking is important since this allows one to estimate the time required for the center temperature to reach a given value. There has been surprisingly little investigation into this problem in the past; in fact, during the last decade and a half, only one group of workers has reported on this topic, Klein *et al.* (1972-81), from the University of Prague, and their reports are not readily available. Prior to their work, Smith *et al.* (1967) presented an analysis of heat transfer from anomalous shapes and applied this work to chilling of hams. The vast majority of the investigations regarding the chilling of roast beef have focused on the question of quality and tenderness. The goal of this study was to determine whether or not one can estimate the time to reach specified temperatures at the center of beef roasts in a relatively straightforward manner. The external cooling conditions are obviously important, because they dictate the rate of cooling of the beef; some industrial processors use very cold water or chilled brine to cool the cooked roasts, while others use cold blast air. The velocity of the cooling medium has a direct influence on the cooling cycle. As is mentioned in a subsequent section, velocities of liquid cooling media (such as ice water) in the range 0.15 to 0.30 m/s will cool a large roast (about 8.2 kg) within the limits specified by FSIS as noted above; while cold air velocities must be significantly higher to achieve the same result.

PROCEDURE

Two sets of data from cooling cooked beef roasts were obtained from commercial meat packers through the Food Safety and Inspection Service (FSIS) of the USDA (Custer 1983). The conditions under which these data were taken are listed below as Cases I and II.

Additional data denoted as Case III, were obtained from cooking experiments conducted to ascertain conditions effective for eliminating *Salmonella typhimurium* from rare dry-roasted beef roasts (Blankenship 1985).

Case I — Cooling with Cold Water

Data on three roasts, 9.0 kg, 9.6 kg and 9.9 kg were supplied by the processor, the approximate roast dimensions given were: 15.25 cm high, 38.9 cm wide, and 43 cm long for the 9.0 kg roast; the same height and width for the 9.6 kg roast but with length 46 cm. The 9.9 kg roast was stated as being 12.7 cm high, 45.7 cm wide, and 48.3 cm long. The roasts were placed in bags impermeable to water and were cooked at a water temperature of 338.7K until the center temperature reached 336.5K. At the end of cooking they were transferred to a rack and immersed in a water cooling tank. The cooling water was maintained at about 272K and was circulated over the roasts at approximately about 0.15 to 0.3 m/s.

Temperatures were recorded by a thermocouple located at the center of the roasts only.

The thermocouple for each roast was attached to a Bristol multipoint recorder, the model number, chart speed range and accuracy were not specified by the packer.

Case II — Cooling with Cold Air

The roast was cooked in an oven at 394.25K until the center reached 333.15K. After completion of the cooking cycle, the bare hot beef roast was placed in a cold air cooler where cold air at a constant velocity and temperature was blown over the unbagged roast. In the case of the roast being considered, the air temperature was maintained between 269.15K and 274.25K but other environmental conditions i.e., humidity, air velocity were not reported by the processor except that the velocity was so high ''it blows my hat off.'' Typical air velocities in these system can range from 1.5 to over 6 m/s; but based on the processors comment a velocity of 4.9 m/s was selected for computation. One thermocouple only of unspecified size was inserted by the processor into the bare roast at the center. The manner of recording the temperature was not given and the temperature of the cooler was stated as being measured within ''about a foot of the roast''. Moreover, the cooler temperature measurement occurred at hourly increments of time and then only for the first seven hours. The roast weight was given as 8.62 kg prior to cooking but overall dimensions were not stated. Based on other roast configurations and in proportion to their weights, dimensions of 15 cm high 39 cm wide and 41 cm long were assumed for this roast.

Case III — Cooling in Still Air

A roast weighing 4.65 kg was cooked in a gas fired oven at 382.86K. The roast weighed 3.45 kg after cooking and was then held at room temperature (294.15K) for 2 h suspended in a cooking net. After the holding period, the roast was stored over night in a cold room at 275.15K to 277.15K. Only the center temperature profile during the 2 h holding period could be compared to prediction because temperature histories were not recorded during cold room storage.

The roast had an elliptic cross section with dimensions given as 12 cm high, 16 cm wide, and 34 cm in length. The corresponding circumference was 44.6 cm as calculated from Eq. (3). Teflon coated iron-constantan thermocouples (36 gauge) were inserted into the roast at 8 cm, 4 cm, and 2 cm, respectively. Surface thermocouples were inserted about one millimeter below the surface for protection aginst the hot gases during the cooking cycle. The thermocouples were calibrated in a water bath at temperatures ranging from 273.15K to 352.55K.

THEORETICAL CONSIDERATIONS

During the cool-down of the cooked roasts of Cases II and III, combined heat and mass transfer effects were possible because of the potential evaporation of water from the surface. Unfortunately, the temperature data available for Case II consisted only of transient temperatures at the center of the beef. Surface and environmental conditions were not specified. It was not possible therefore to estimate whether or not a "constant drying rate" occurred during cool-down; nor was it possible to detect any changes in temperature at the center of the roast due to surface evaporation. In Case III, although transient surface temperatures were available, there was no experimental indication that a constant temperature occurred during cool-down. Moreover, details regarding environmental conditions were not available. It was decided therefore to approach the prediction of the center temperature of the roasts during cool-down in Cases II and III by heat transfer means alone. This approach will be discussed now for all three cases.

Case I — Cooling with Cold (@ 273.15 K) Water

Cooling the roasts with cold water at a velocity of ca. 0.15 m/s results in little surface resistance in contrast to air cooling.

Consequently, the solution given by Kirkpatrick and Stokey (1959) for transient heat transfer in elliptical plates and cylinders may be used to predict the center line temperatures. Their solution is based on McLachlan's (1945) approach who expressed the heat equation in Cartesian coordinates

$$\frac{\partial u}{\partial t} = \alpha \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{1}$$

Using elliptic coordinates and separation of variables, one obtains Mathieu ordinary differential equations which are solved by Kirkpatrick and Stokey for the case of step changes in the surface temperature. The solution to the problem in which the elliptic cylinder is initially at uniform temperature and the surface temperature is suddenly changed to a new value, involves a double series sum-

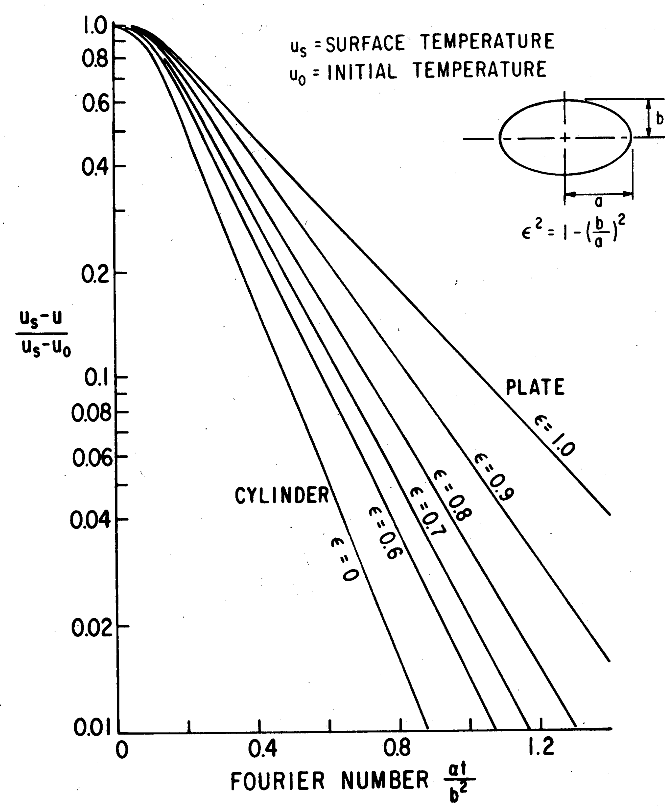

FIG. 1. CENTRAL TEMPERATURE HISTORIES OF ELLIPTICAL CYLINDERS OF ECCENTRICITIES 0.0, 0.6, 0.7, 0.8, AND 0.9, 1.0

mation and is quite complex. Details are given by Kirkpatrick and Stokey. The solution for the center line temperature is presented as a graph (Fig. 1).

The convective heat transfer to the beef was computed using the Colburn (1933) "j" factor analogy between fluid friction and heat transfer (see also Perry 1973)

$$0.664\ (N_{Re})^{-1/2} = j_{AVG} = N_{st}(N_{Pr})^{2/3} \qquad (2)$$

This relation is chosen because of the relative "squatness" of the beef being cooled (15.2 cm high, 38.9 cm wide and 43 cm long for the 9.0 kg roast; the same height and width for the 9.6 kg roast but with a length of 46 cm. The 9.9 kg roast was 12.7 cm high, 45.7 cm wide and 48.3 cm long) and the orientation of the roasts to the cold water (perpendicular to the longitudinal axis) as indicated by the processor. The thermal properties of the cooked beef were obtained from three sources, Baghe-Khandan *et al.* (1981, 1982) Morley (1972) and Ordinanz

(1946). Representative compositions of cooked beef were taken from Watt and Merrell (1975); which suggest that the water content is approximately 62% after cooking. Baghe-Khandan *et al.* show that the thermal conductivity of cooked beef is essentially constant over the temperature range 40°F to 150°F at 0.39 W/mk (0.23 BTU/h ft °F). The specific heat data listed by Morley (1972) was compiled from the work of Ordinanz (1946) and is given as constant over the temperature range 273.15K to 373.15K as equal to 3054 J/kg K (0.73 BYU/lb °F).

Density data were taken from the 1982 work of Baghe-Khandan *et al.*; these data together with the compositions listed by Watt and Merrell give a density of 1105 kg/m³ (69 lb/ft³).

In Eq. (2) the properties of the fluid are to be evaluated at a "film" temperature given by ½ $(u_w + u_b)$, the average of the beef surface temperature u_w and the bulk fluid temperature u_b.

The beef surface temperatures vary during cool-down and hence the surface temperature u_w was chosen as the log. mean average of the final and initial beef temperatures for use in calculating the "film" temperature. The surface temperature history was not recorded by the processor; however, it is known that the roasts were cooked at an environment temperature of 340.15K. (The corresponding final center temperatures were 335.35K, 336.45K, 337.55K for the 9.0 kg, 9.6 kg and 9.9 kg roasts, respectively). An initial surface temperature of 336.16K was selected therefore to allow for a reasonable decrease in surface temperature during the time of transfer from cooking to cooling cycle. The corresponding final surface temperature was chosen as 274.25K since the surface temperature very nearly approaches the cooling water temperature after a few hours of cooling.

Comparison between measured and estimated temperatures are shown in Fig. 2 for the three roasts.

Case II — Cooling by Cold Air

Prediction of the center temperature in Case II requires that one consider the finite surface resistance because of the use of cold air as a cooling medium. There is no known analytic solution in elliptic cylindrical coordinates for which the center temperature may be computed if a boundary resistance is present.

Consequently the elliptic cylinder configuration was replaced by an equivalent rectangular slab having the same surface area. The circumference of an ellipse "S" is given in terms of the major semi-axis "a" and eccentricity ϵ by the converging series (Bartsch 1974):

$$S = 2\Pi a \left[1 - (\tfrac{1}{2})^2 \epsilon^2 - \frac{(1\cdot 3)^2}{2\cdot 4} \frac{\epsilon^4}{3} - \frac{(1\cdot 3\cdot 5)^2}{2\cdot 4\cdot 6} \frac{\epsilon^6}{5} - \ldots \right] \quad (3)$$

$$\text{where } \epsilon^2 = 1 - \left(\frac{b}{a}\right)^2 \quad (4)$$

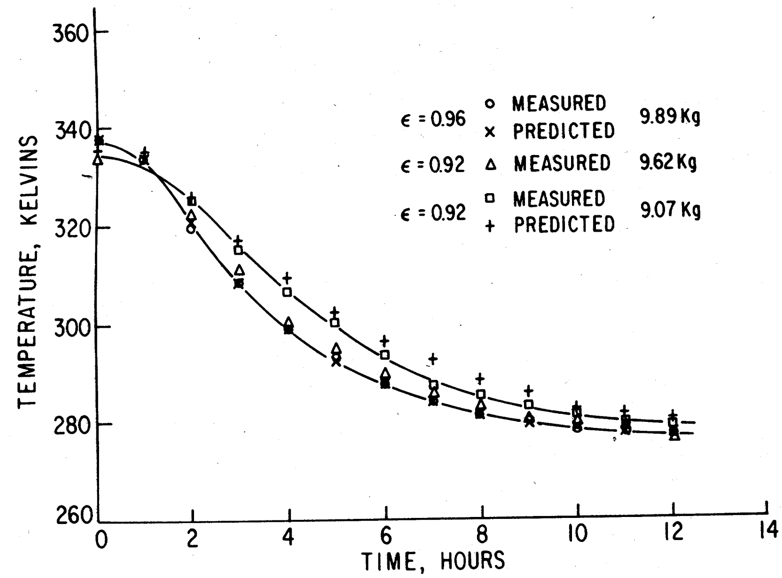

FIG. 2. PREDICTED AND MEASURED CENTER TEMPERATURES OF COOKED BEEF ROAST DURING COLD WATER COOLING

The dimensions of an equivalent rectangular slab are therefore set by the value of the circumference of the ellipse calculated from Eq. (3). An equivalent rectangular width of 30 cm with height 15.2 cm results.

The Colburn (1933) relation for fluids flowing in turbulent flow across a flat plate was selected for obtaining an estimate of the convective heat transfer coefficient:

$$j = \frac{h}{\rho C_p V} (N_{pr})^{2/3} = 0.036 \, (N_{Re})^{-0.2} \tag{5}$$

where the transport properties are to be evaluated at a "film" temperature equal to $\frac{1}{2}(u_w + u_b)$. An estimate of the wall temperature was made by computing a log mean temperature difference between a "starting" surface temperature and the temperature at the end of the cooling period.

A contribution for radiation in the cold locker was included in the estimation of the overall heat transfer coefficient.

The simplification suggested by McAdams (1957) for temperatures reasonably close together was used i.e.

$$h_r = \sigma \, \epsilon \, \frac{(u_1^4 - u_s^4)}{(u_1 - u_s)} \tag{6}$$

This assumes that the cold locker surroundings are at about 273.15K with the initial average surface temperature 344.4K and final surface temperature of 277.8K. A total hemispherical emissivity of 0.9 has been used for this case. (General Electric Company Heat Transfer Data Book 1973).

The effective h_r value was calculated by taking the average of the coefficients calculated at the beginning of cool down when u_1 was 344.4K, (with $u_s = 273.15$) and at the end when u_1 was 277.8K (and u_s 273.15K). i.e.

$$h_{r_{AVG}} = (5.67\ (0.9)\ 10^{-8}\ \frac{1}{2} \left\{ \frac{(u_1^4 - u_s^4)}{(u_1 - u_s)}_{start} + \frac{(u_1^4 - u_s^4)}{(u_1 - u_s)}_{end} \right\} \tag{7}$$

As noted before there is no known analytic solution to the heat equation in elliptic cylindrical coordinates if a finite boundary resistance is present; but selection of an equivalent rectangular geometry allows one to use the known solution (Carslaw and Jaeger 1959)

$$u = u_\infty + 2\sum_{n=1}^{\infty} e^{-\alpha\lambda_n^2 t} \left\{ \frac{[H^2 + \lambda_n^2] \cos(\lambda_n X)}{(L[\lambda_n^2 + H^2] + H)} \right\} \int_0^L (f(x) - u_\infty) \cos(\lambda_n x) dx \tag{8}$$

$$\text{Where } \lambda_n \text{ are eigenvalues satisfying } \lambda_n \sin(\lambda_n L) = H \cos(\lambda_n L) \tag{9}$$

Computation of the center temperature however presents a difficulty in this case; the initial temperature history f(x), after cooking and prior to cold air cooling is not known because it was not measured. It was decided, therefore, to attempt to predict the center temperatures by comparing a calculated value with the measured temperature assuming various constant temperatures for f(x) in (8). Of all the values selected for f(x) in Eq. (8) it was found that f(x) = 333.15 gave the best comparison. Estimated and measured temperatures are shown in Fig. 3.

Case III — Cooling in Still Air (295.15K)

Cooling in quiescent air interjects a significant boundary resistance to heat transfer. As in previous Case II, no known analytic solution exists in elliptic coordinates when the surface resistance is finite. Estimation of the heat transfer coefficient was made using two approaches:

(1) A cylinder of equal length as the cooked roast with a diameter (14.2 cm) was selected giving an equivalent circumference as the roast. (44.6 cm).

(2) A horizontal slab model was used of equal length as the cooked roast with dimensions giving the same perimeter as the roast. A height of 12 cm and width of 10.3 cm results if the height is chosen to match the semi-minor axis. There is no combination of dimensions which will match both the perimeter and cross section area of the ellipse.

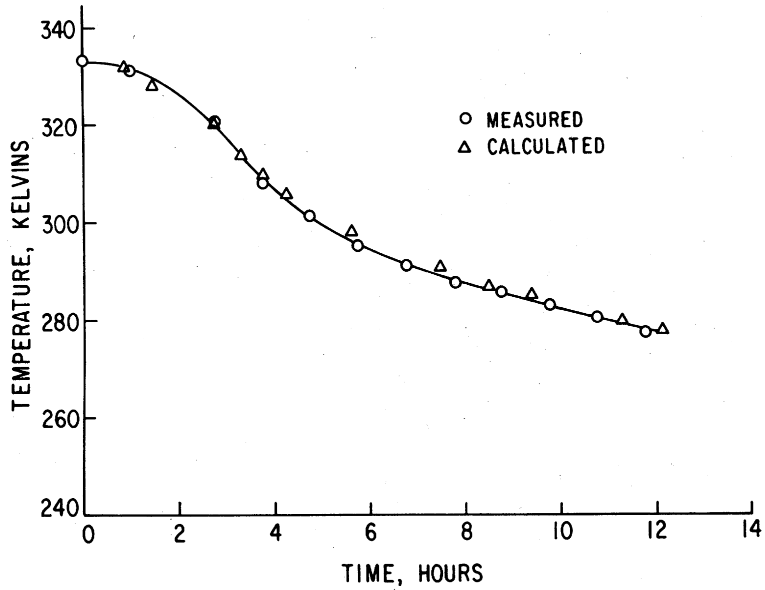

FIG. 3. PREDICTED AND MEASURED CENTER TEMPERATURES OF COOKED ROASTS DURING COLD AIR COOLING
Air velocity ca. 4.9 m/s. Roast weight: 8.6 kg before cooking.

Heat transfer was considered by combined natural connection and radiation. In the estimation of the radiation contribution an effective heat transfer coefficient h_r was used, (Eq. 6 and 7) as in Case II (McAdams 1954).

The average value of h_r was estimated based on the start and finish of the cookdown cycle i.e., a value based on an initial surface temperature of 367K radiating to the surroundings, 294.4K and the final surface temperature 309.8K radiating to the surroundings 294.4K. As in Case II, a total hemispherical emissivity of 0.9 was used (General Electric Heat Transfer and Fluid Flow Data Book 1973).

This results in an estimated heat transfer coefficient of 6.4 W/m^2K; an estimate of the natural convection coefficient was made by using the simplified form for vertical cylinders less than 0.9 m air suggested by King (1932) as discussed in Jacob (1949) i.e., $h_c = 1.44(\Delta u/L)^{1/4}$ W/m^2K where Δu is the difference between the log mean surface temperature (337.6K) and the surrounding (294.26K). The contribution of the radiation and natural convection yields an estimated overall effective heat transfer coefficient of (6.4 + 4.8) W/m^2K or 11.2 W/m^2K. The corresponding Biot number for the cylinder is 1.9.

Solutions for the center temperature of the cylinder and slab are known (Carslaw and Jaeger 1959).

The temperature at the center of a cylinder with a finite surface resistance and initial temperature gradient f(r) at time equal to zero is

$$u_{(0,t)} = u_\infty + \frac{2}{b^2}\sum_{n=1}^{\infty}\left\{\frac{e^{-\alpha\lambda_n^2 t}\lambda_n^2}{[H^2 + \lambda_n^2]\, J_0^2(\lambda_n b)}\right\}\int_0^b r[f(r) - u_\infty]J_0(\lambda_n r)dr \quad (10)$$

here the boundary condition at the surface $r = b$ is $(\partial u/\partial r) = H\,(u - u_\infty)$. The

$$\lambda_n \text{ eigen-values satisfy } \lambda_n J_1(b\lambda_n) = HJ_0(b\lambda_n); \quad (11)$$

values are given by Carslaw and Jaeger 1959.

An initial quadratic temperature profile resulted from a curve fit of the center temperatures in Fig. 4. This profile reduces to the form $f(r) = A + mr^2$ by the constraint that at $r = 0$, $\partial u/\partial r = 0$. Completing the integration in Eq. (10) leads to:

$$u_{(0,t)} = u_\infty + 2\sum_{n=1}^{\infty} \xi_n e^{-\xi_n^2\alpha t/b^2}\,\frac{([A + mb^2 - u_\infty]\, J_1(\xi_n) - 2mb^2/\xi_n\, J_2(\xi_n))}{\left[\left(\frac{hb}{k}\right)^2 + \xi_n^2\right] J_0^2(\xi_n)} \quad (12)$$

where $\xi_n = (\lambda_n b)$ and J_0, J_1, J_2 are Bessel functions of the first kind of order zero, one and two, respectively.

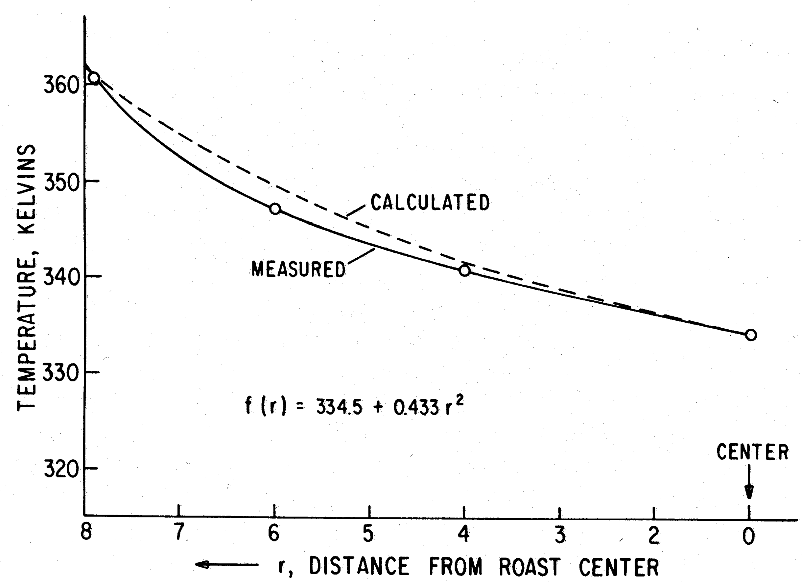

FIG. 4. TEMPERATURE PROFILE OF ROAST BEEF AT THE CENTER EXPOSED TO AMBIENT CONDITIONS (294.26k) JUST AFTER REMOVAL FROM OVEN

Attempts to predict the measured temperature at the center by using the one-dimentional slab model with a prescribed quadratic temperature profile were not successful, Fig. 5; these predictions are given by Eq. (13).

$$u_{(0,t)} = u_\infty + 2 \sum_{n=1}^{\infty} e^{-\xi_n^2(\alpha t/L^2)} \xi_n^2 \frac{[\xi_n^2 + (hL/k)^2]}{[\xi_n^2 + (hL/k)^2 + (hL/k)]}$$

$$\left\{ (A + mL^2 - u_\infty)(hL/k) - 2mL^2 \left[\frac{(hL/k)}{\xi_n^2} - 1 \right] \right\} \cos(\xi_n) \qquad (13)$$

$$\text{here, } \xi_n \tan\xi_n = (hL/k) \qquad (14)$$

Results of these estimates are given in the next section.

RESULTS

Case I — Cooling with Cold (0°C) Water

The calculated Reynolds numbers when cooling the 9.0 kg, 9.6 kg, and 9.9 kg, roasts were 42,024, 42,024 and 49,371 respectively; an estimated film temperature of $\frac{1}{2}(u_w + u_b)$ equal to 280.75K was used to evaluate the transport properties. The heat transfer coefficients computed from Eq. (2) were 490 W/m²K (86BTU/h ft°F) for the 9.0 and 9.6 kg roasts and 452 W/m²K for the 9.9 kg roast. Laminar flow was assumed in all cases; it is known (Eckert and Drake 1959) that at a Reynolds number of approximately 50,000 turbulence occurs in flows across a flat plate. The Biot numbers which resulted from using the above heat transfer coefficients with the shortest heat transfer path i. e. (semi-minor axis) for the roasts were 93, 93 and 72, respectively. The eccentricities of these roasts were 0.92, 0.92 and 0.96 based on the aforementioned dimensions supplied by the processor and hence were almost equivalent to a flat plate. Prediction of the center temperatures were made using Fig. 1. The results plotted in Fig. 2 show fairly good agreement in all cases, the largest deviation between predicted and measured values being about 6% (at 6 h). The total time of cooking, 12 h, to 277.55K (40F) is seen to be predicted satisfactorily. The center temperatures are also predicted very satisfactorily by the method of Smith *et al.* (1967).

Case II — Cooling with Cold Air

Surface Temperature Selection. In the absence of measured surface temperature data for this particular roast, an overall starting temperature of 344.25K was selected based mainly on the surface temperature histories obtained in Case III. (The roast was cooked at 391.15K). The final temperature chosen, 277.55K,

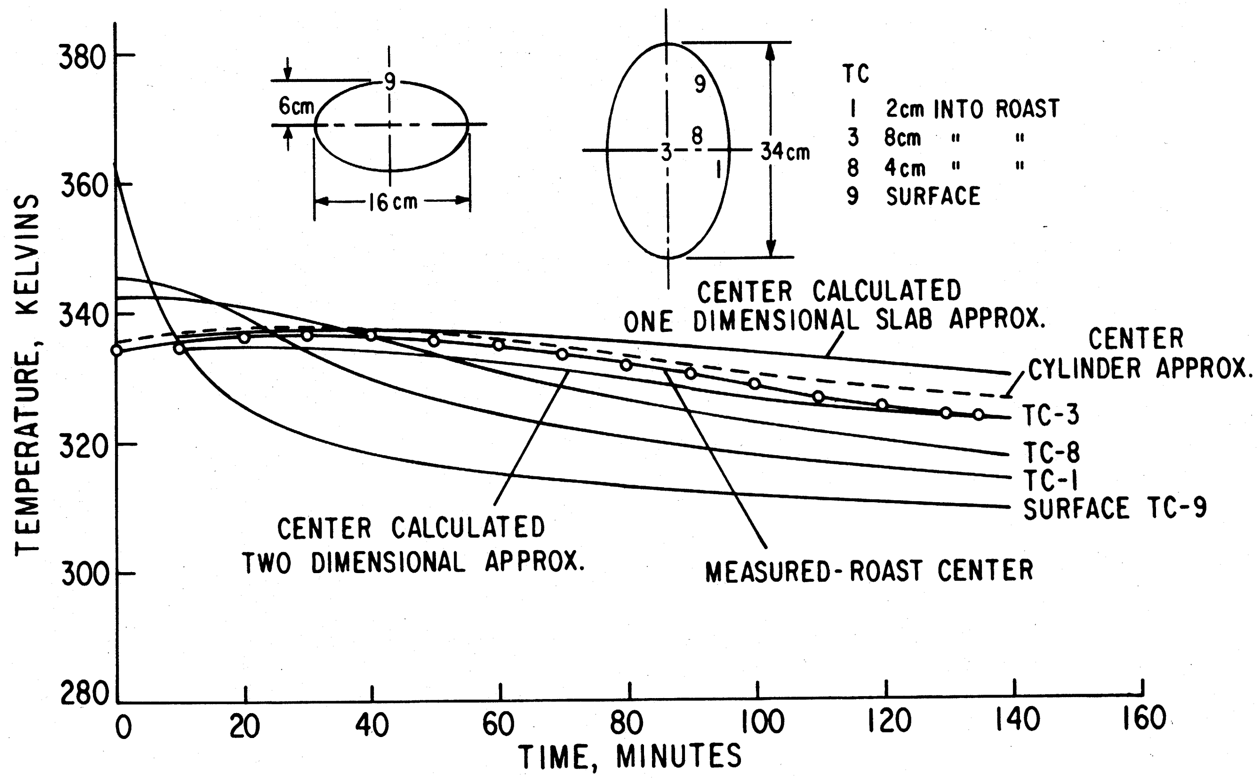

FIG. 5. TEMPERATURE PROFILE OF ROAST BEEF EXPOSED TO AMBIENT CONDITIONS 294.26K (70°F) AFTER COOKING IN OVEN AT 383.15K (230°F) Comparison of predicted and measured values.

together with the above estimated starting temperature gave a log mean surface temperature of 309.7K; a corresponding "film" temperature of 290.K was estimated based on a cold air temperature of 207.4K.

Heat Transfer Coefficient. An estimated convective heat transfer coefficient of 27. W/m^2K (4.8 BTU/h ft_2°F) was calculated from Eq. (2) using an Reynolds number of 98,650. An estimate of the radiation contribution from the walls of the cooler assumed to be at 272K gave an effective radiation coefficient h_r of 5.2 W/m^2K. The overall coefficient, 32.2 W/m^2K gives a Biot number of 6 by employing the shortest heat transfer path $\left(\frac{15.2}{2}\right)$ cm.

Center Temperature Comparison. When comparing the calculated center temperatures with measurement, it was found that selection of an initial temperature for f(x) in Eq. (8) was very sensitive to the calculated results, as one might expect.

It was found that by selecting f(x) = 333.15K as the initial "average" temperature the best comparison of calculated center temperatures with measurement resulted, the greatest difference (5K) occurring at time 2.8 h, all other differences were within 1 °K Fig. 3. The selection of f(x) = 338.75K, for example, resulted in differences between calculated and measured temperatures of about 6.1 °K for the first 6 h of cooling; afterward these temperature differences reduced to between 3.3K and 1.7K as shown in Fig. 3. Curiously, at time equal to 2.8 h the calculated and measured temperature agreed to with 1K.

Selection of an equivalent cylinder to represent the roast and using the calculated quadratic profile shown in Fig. 4 gave fair agreement with measurement. These temperatures were calculated using Eq. (12). As shown in Fig. 5, the calculated temperatures are about 1.7K degrees above the measured values. The predicted temperatures at the end of cooling are about 3.3K degrees above the experimental value.

It should be noted that the true surface temperature was probably somewhat above that recorded by thermocouple 9, Fig. 5. Thermocouple 9 was inserted 1 to 2 mm below the surface of the beef. Rough extrapolation shows that the surface temperature could have been 5 to 6K higher than those recorded i. e. 371K to 373K, which would have an influence on the above result.

Prediction of the center temperature by using a one-dimensional slab model with a prescribed quadratic temperature profile was not totally successful, Fig. 5. During the first hour of cooling, agreement between prediction and measurement is fairly satisfactory; beyond this time however, digression occurs. The final difference in temperature between measurement and prediction using the horizontal slab model is 5.5 K.

The two dimensional computation using a horizontal slab and the product solution method (Newman 1936; Carslaw and Jaeger 1959) appears to give fair agreement with measurement, Fig. 5. The agreement is somewhat deceptive because the product solution procedure requires the initial temperature to be a constant. The experimental information necessary to obtain axial temperature profiles in the vertical and horizontal directions was absent so in the case of the two dimensional calculation a temperature of 334.26K was chosen arbitrarily as the initial temperature. It would seem more logical to select some log mean difference between the initial surface temperature and the center temperature; which value to choose is not obvious. A reasonable selection is hampered because the surface temperature was not quite uniform around the circumference of the beef at the end of the heating period.

Selection of an equivalent cylinder to represent the roast and using the calculated quadratic profile shown in Fig. 4 gave fair agreement with measurement.

CONCLUSIONS

(1) When cooked roast beef is chilled by circulating ice water or a fluid at approximately 273K (32 °F), fairly accurate prediction of the center temperature history may be made by employing the graph in Fig. 1 (or by using the method of Smith, Nelson and Henrickson (1967)) It appears that if the velocity is reduced to about 0.03 m/s the resulting Biot number (ca. 30) suggests the presence of a boundary layer resistance.

(2) The use of cold air to cool cooked roasts introduces a thermal resistance at the surface thereby making it more difficult to predict a center thermal history compared to (1) above. Moveover, the surface temperature tends to be nonuniform during cool-down as a consequence of the flow field surrounding the beef. This complicates thermal history prediction and might interfere with a time-temperature requirement for the inactivation of certain organisms.

Depending upon the geometric configuration of the beef it may be possible to estimate the center temperature history by using either a horizontal rectangular slab or a solid cylinder model. If the beef eccentricity is 0.5 to 0.70, a cylindrical model is reasonable, from 0.85 to 1.0, a plate is the model of choice. Intermediate values, i.e., 0.7 to 0.85, do not lend themselves to reasonable temperature prediction (i.e., within 5.5K) with either a pure cylinder or flat plate model. It is probably best to use a numerical method for center temperature prediction in this range.

It should be clear that when cooling large roast ca. 7.3 to 9 kg, air is much less efficient than ice water as a heat transfer medium. The heat transfer coefficient of water and air can vary by more than a factor of 10 at the same flow velocity over similar objects.

(3) Prediction of the center temperature profile of a cooked roast in the case of quiescent air is difficult. In addition to possible mass transfer effects, radiation and natural convection contribute equally to the cooling process. In the case of small roasts a reasonable (within 5.5K) estimate of the temperature history may be made by employing a cylindrical heat conduction model for the roast. This choice, however, is sensitive to the eccentricity of each roast; above an eccentricity of about 0.85 a flat plate model is more appropriate. Determination of the magnitude of the "overshoot" at the onset of cooling can be made, however, only if the initial temperature history is known. In the model described herein, notice (Fig. 5) that the magnitude and length of time of the "overshoot" is predicted quite accurately.

NOMENCLATURE

A = constant in Eq. (12)
a = major semi-axis of ellipse; constant in Eq. (4)
b = minor semi-axis of ellipse; radius of cylinder
Bi = Biot number; hL/k
C_p = specific heat at constant pressure
h = heat transfer coefficient
H = h/k
J = Bessel function of first kind
j = Colburn factor analogy between heat and momentum transfer
k = thermal conductivity
L = thickness
N_{Bi} = Biot number
N_{Re} = Reynolds number
N_{st} = Stanton number $h/\rho VC\rho$
N_{pr} = Prandtl number
r = radial coordinate
S = circumference of ellipse
t = time
u = temperature
V = velocity
W = power
x,y = position condinates

Greek Letters

α = thermal diffusivity
ϵ = eccentricity of ellipse
λ_n = eigenvalues Eq. (8)(10)
ξ_n = eigenvalues Eq. (12)
ρ = density

NOMENCLATURE *(continued)*

Subscripts

AVG = average value
b = bulk value
c = convective
f = film
n = counting index
o = initial condition
r = radiation
s = surroundings temperature
w = wall
∞ = ambient condition

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